Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education
Sample Examination, 2016

Question/Answer Booklet

MATHEMATICS APPLICATIONS

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable):

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

2013/42074

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **65% (96 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(7 marks)**

A ball is dropped from a height of 60 cm onto a horizontal surface. The height reached by the ball after each bounce is two-thirds of the height of the previous bounce.

(a) Write a recursive rule to show the distance travelled by each successive downward motion of the ball. (2 marks)

(b) What is the maximum height reached by the ball after the third bounce, correct to **two** decimal places? (2 marks)

The coefficient of restitution e is a measure of the bounciness of an object such as a ball on impact with the floor. This can be measured by the ratio

$e = \frac{H_{n+1}}{H_n}$ where H_n= the height of a given bounce and $0 \leq e < 1$

(c) (i) Calculate the coefficient of restitution for the ball in part (a). (1 mark)

(ii) Given $e = 0.75$ calculate to the nearest centimetre, the maximum height of the third bounce of a ball dropped from 2 metres. (2 marks)

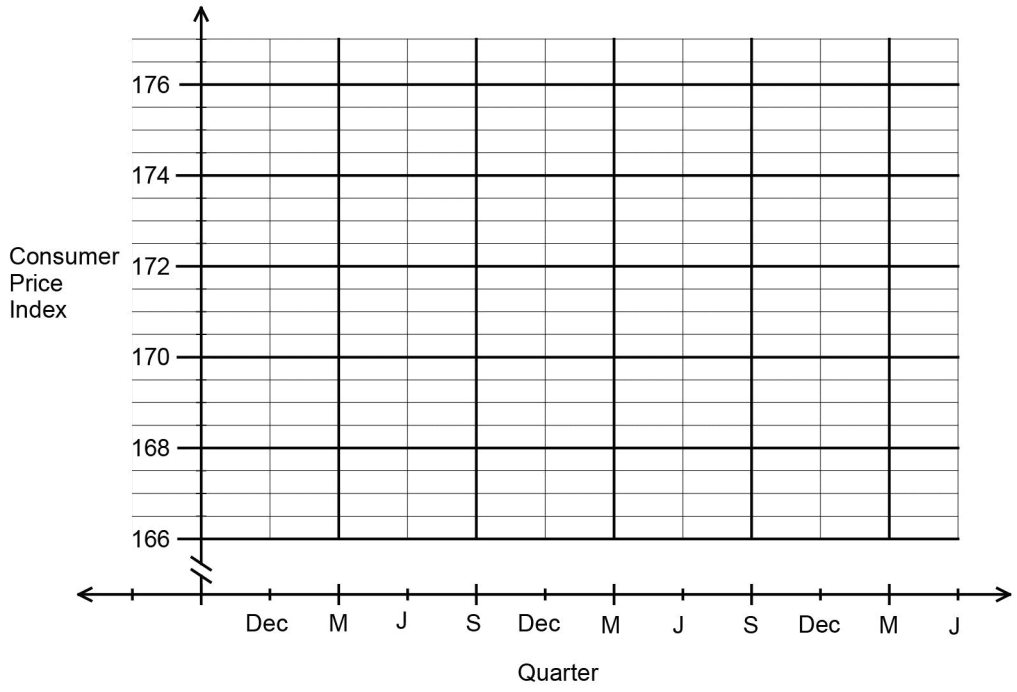
See next page

Question 10 **(7 marks)**

The Consumer Price Index (CPI) is measured every three months and is used to help determine inflation rates. The following information has been provided by the Australian Bureau of Statistics.

Quarter	**Dec 2008**	**Mar 2009**	**June 2009**	**Sept 2009**	**Dec 2009**	**Mar 2010**	**June 2010**	**Sept 2010**	**Dec 2010**	**Mar 2011**	**June 2011**
Month (m)	1	4	7	10	13	16	19	22	25	28	31
Consumer Price Index (i)	166.0	166.2	167.0	n.a.	169.5	171.0	172.1	173.3	174.0	n.a.	n.a.

(a) Plot the above data on the axes below. (2 marks)



(b) Determine the equation of the least squares line that models the relationship between the time in months and the CPI for the data provided. (1 mark)

(c) Use your line from part (b) to predict the CPI for September 2009. (1 mark)

(d) Which would be the more accurate prediction: The CPI for September 2009 or the CPI for September 2011? Justify your choice of answer. (2 marks)

(e) The data show an increasing trend in the value of the CPI over time. What does the equation, as determined in part (b), indicate is the rate of increase in the CPI? (1 mark)

See next page

Question 11 **(8 marks)**

(a) The results of a survey of students' first preferences for work placements are given in the table below.

	Monday	Tuesday	Wednesday
Kindergarten	42	48	43
Hospital	23	24	26
Nursing home	14	12	13

(i) If Kindergarten placements are allocated to Mondays, Hospital placements are allocated to Tuesdays and Nursing home placements are allocated to Wednesdays, how many students will receive their first preferences? (1 mark)

(ii) Identify the maximum number of students who can receive their first preference by reallocating each work place to a different day of the week. (1 mark)

(b) An extra work place has become available. Thursdays were included in the roster and a new survey for first preferences was conducted with the results shown in the table below.

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	23	21	19	13
Hospital	21	19	18	10
Nursing home	13	13	9	12
Day care centre	15	15	19	13

See next page

The Hungarian algorithm is to be used to identify the maximum number of students who could get their first preference by allocating each work place to a particular day of the week.

(i) The first step of the algorithm has been partially completed. Complete the table below to show the result of this process. (1 mark)

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	0	2	4	10
Hospital	2	4	5	13
Nursing home	10	10	14	11
Day care centre				

At a later stage of the Hungarian algorithm, the table was presented as follows:

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	0	2	4	9
Hospital	0	2	3	10
Nursing home	0	0	4	0
Day care centre	4	4	0	5

(ii) Complete the next step of the algorithm in the table below (3 marks)

	Monday	Tuesday	Wednesday	Thursday
Kindergarten				
Hospital				
Nursing home				
Day care centre				

(iii) Allocate the work places to the correct days to maximise the first preferences by shading the correct cells in the table below. (2 marks)

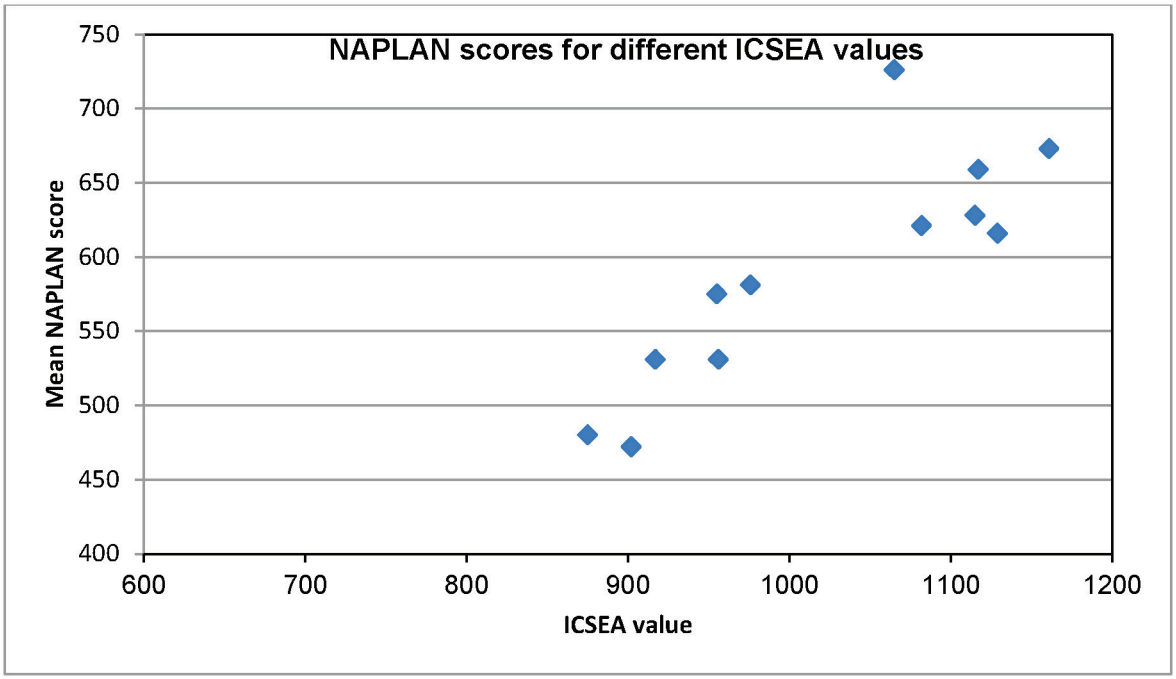
	Monday	Tuesday	Wednesday	Thursday
Kindergarten	23	21	19	13
Hospital	21	19	18	10
Nursing home	13	13	9	12
Day care centre	15	15	19	13

Question 12 **(5 marks)**

The following figures were obtained from the My School website. The second column shows the ICSEA value for the school (an indication of the socioeconomic circumstances of the school population) and the third column shows the mean score for the school on the 2008 NAPLAN numeracy test for Year 9 students in that school.

School	ICSEA value (x)	Mean NAPLAN score Year 9 Numeracy (y)
A	1117	659
B	1082	621
C	875	480
D	955	575
E	1129	616
F	917	531
G	1065	726
H	976	581
I	902	472
J	956	531
K	1115	628
L	1161	673

From the scatterplot of the data:



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(a) (i) Identify the response variable. (1 mark)

(ii) Describe the apparent relationship between the two variables. (1 mark)

(b) Calculate the correlation coefficient r_{xy}. (1 mark)

(c) Calculate the coefficient of determination. (1 mark)

(d) What percentage of the variation in mean NAPLAN scores between these schools can be explained by the variation in ICSEA values? (1 mark)

See next page

Question 13 **(11 marks)**

To save money to travel overseas, Luke started an investment account. He used $1000 to open the account and then deposited an extra $200 at the end of each month for two years.

The table below shows the following:

- The amount in the account at the beginning of each month (A_n)
- The interest added to the account each month (I_n)
- The deposit made near the end of each month (D_n)
- The amount in the account at the end of each month (A_{n+1}).

Month (n)	Amount at beginning of month (A_n)	Calculation of interest for month (I_n)	Deposit for month (D_n)	Amount at end of month (A_{n+1})
1	$1000.00	$1000.00 × 1.012	$200.00	$1212.00
2	$1212.00	$1212.00 × 1.012	$200.00	$1426.54
3	$1426.54	$1426.54 × 1.012	$200.00	$1643.66
4	$1643.66	$1643.66 × 1.012	$200.00	$1863.39
5	$1863.39	$1863.39 × 1.102	$200.00	$2085.75
6	$2085.75	$2085.75 × 1.012	$200.00	$2310.78

Note: The values in this table have been rounded to two decimal places.

(a) What is the monthly interest rate? (1 mark)

(b) Write a recursive rule to calculate the amount in the account at the end of each month. (2 marks)

(c) How much is in the account after two years? (2 marks)

See next page

(d) How much interest did Luke receive over the two-year period? (2 marks)

(e) For many investment accounts, interest is calculated daily but paid into the account monthly. Would this process have given Luke a higher or lower amount of interest over the two years? Explain your choice of answer. (2 marks)

(f) If Luke had been offered double the rate of interest but still opened his investment account with $1000 and deposited $200 each month would the interest earned on his investment over the two-year period have been:

A twice as much
B less than twice as much
C more than twice as much.

State your choice and give a reason for your answer. (2 marks)

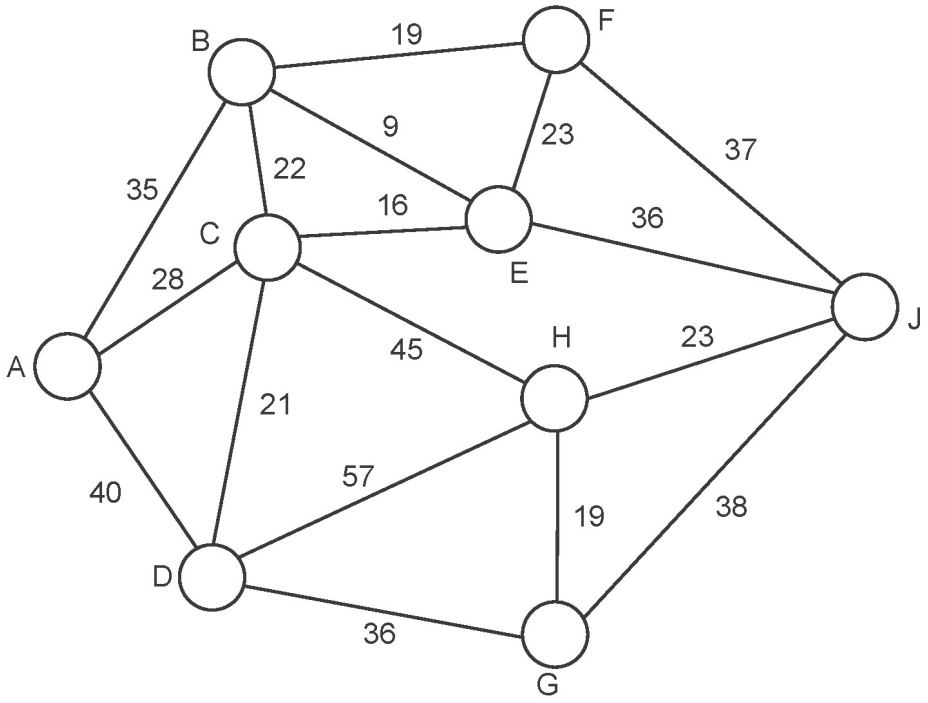
See next page

Question 14 **(8 marks)**

A local park has nine main attractions. The paths between these attractions are shown on the network below, together with the distances, in metres, between each attraction.

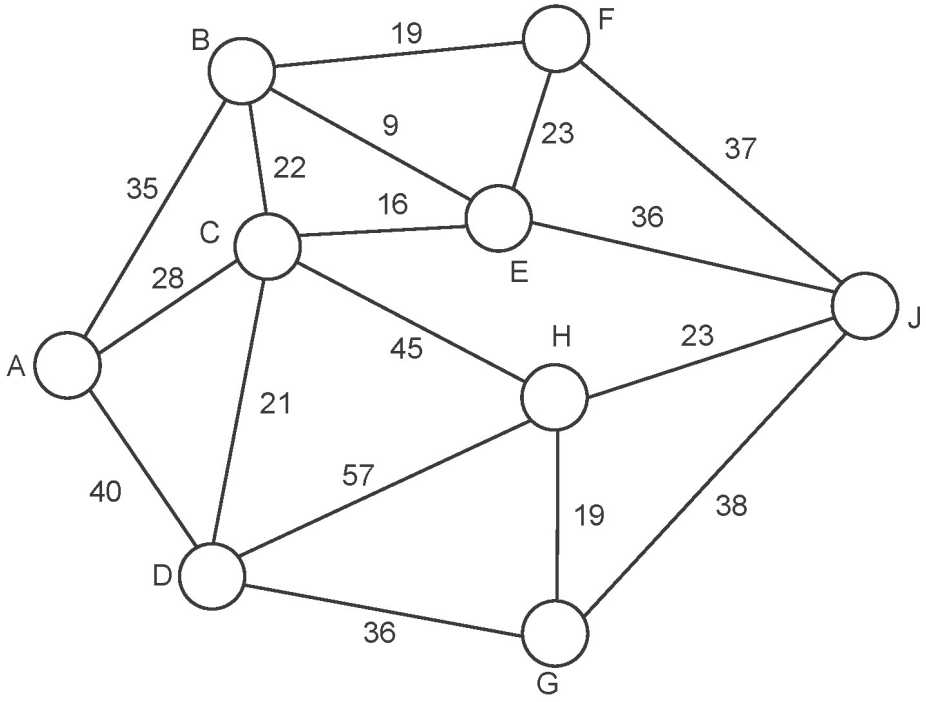
Jason is at the children's playground A and Sandra is at the barbecue area D. Sandra calls Jason on his mobile phone and says that she wants to go with him to the waterfall J. She will wait for him at the barbecue area D.

(a) What is the shortest path for Jason to travel from the children's playground A to the waterfall J, joining Sandra at the barbecue area D on the way? State both the path and the distance. (3 marks)



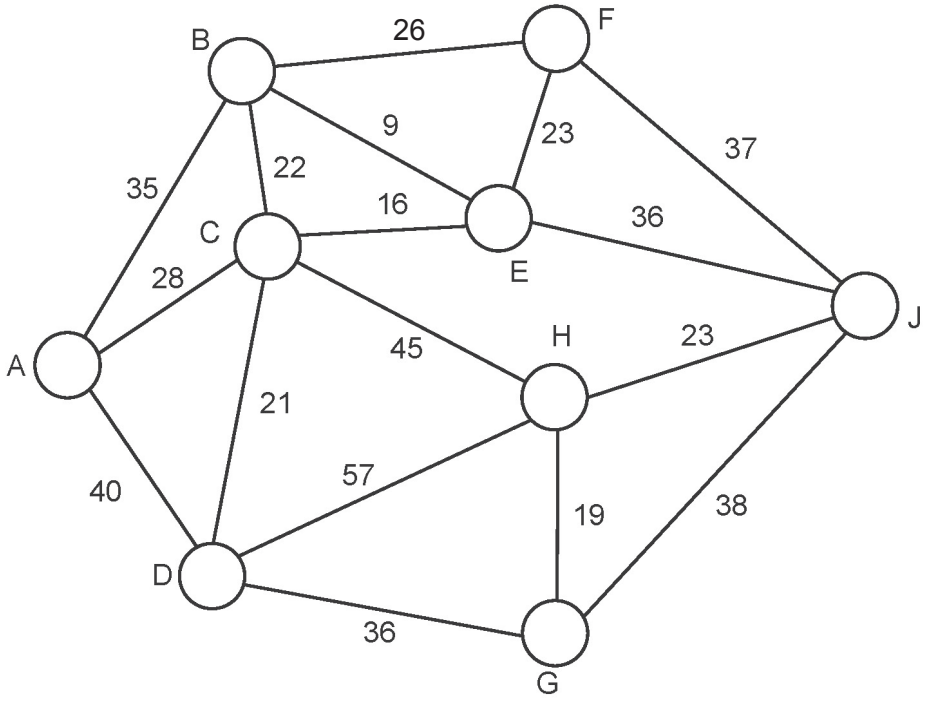
(b) The local council has decided to resurface some of the connections between the nine attractions. They want every attraction to be connected so that wheelchairs can move between them. The new connections will be expensive to resurface and the cost is to be kept to a minimum.

(i) On the diagram below show the connection(s) that the council should resurface. (2 marks)



(ii) What is the total length of the connections to be resurfaced? (1 mark)

(c) Before the resurfacing began, the connection between attractions B and F had to be altered to 26 m in length. How did this alteration change the connections to be resurfaced and the total length? (2 marks)



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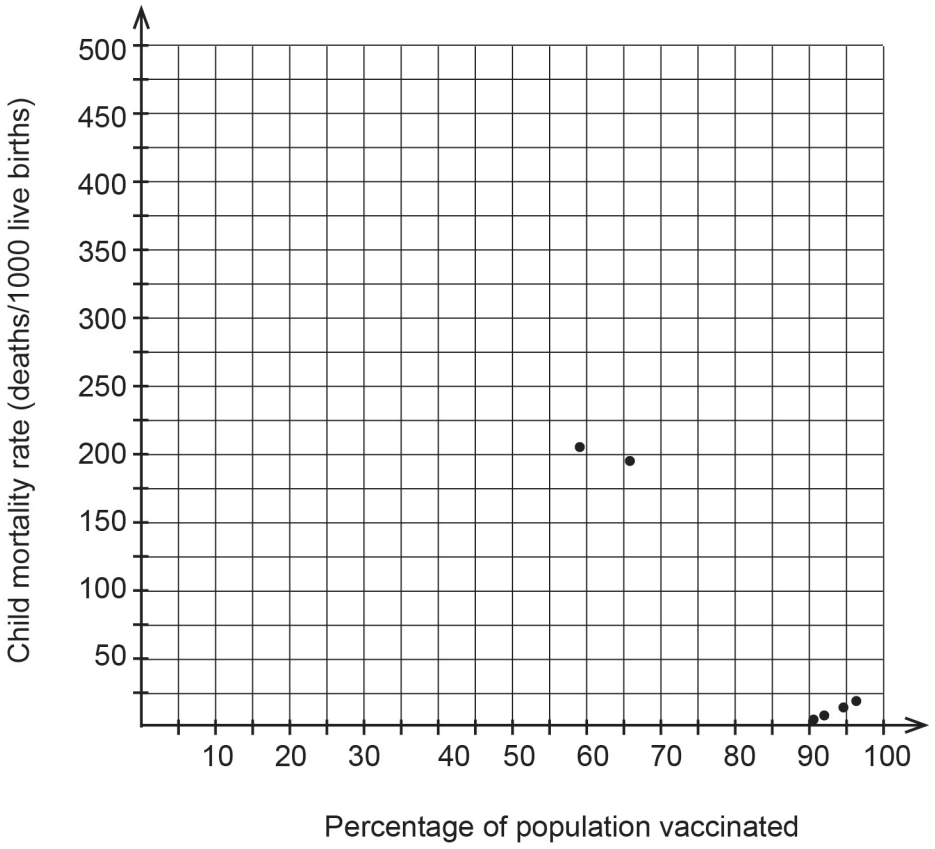
Question 15 **(6 marks)**

The World Health Organisation (WHO) monitors the percentage of a country's target population that has been vaccinated against a number of diseases as well as the child (under 5 years) mortality rate for that country. The table below shows the percentage of the target population vaccinated against hepatitis B and the child mortality rate (deaths per 1000 live births) for 12 countries (A–L).

Country	Percentage of target population vaccinated against hepatitis B (p)	Child mortality rate (deaths/1000 live births) (m)
A	92	5
B	66	199
C	59	209
D	94	15
E	96	21
F	90	4
G	83	84
H	56	68
I	66	138
J	73	191
K	64	112
L	83	39

(a) The information for the first six countries (A to F) has been plotted on the scatterplot below. Complete the scatterplot. (2 marks)



See next page

The equation for the least-squares line that models the relationship between the percentage of the target population vaccinated (p) and the child mortality rate (deaths per 1000 live births) (m) is $m = -4.0808p + 403.959$ and the correlation coefficient is $r_{pm} = -0.7565$.

(b) Predict the child mortality rate where only a quarter of the target population has been vaccinated. (2 marks)

(c) Comment on the reliability of your prediction from part (b). Give one reason to justify your answer. (2 marks)

Question 16 **(7 marks)**

Karen borrowed \$20 000 to purchase a new car. Interest on the loan was set at 8.5% per annum. If the interest of the loan ($\$A$) is compounded n times per year, the amount owed ($\$V$) after one year is given by $V = A\left(1+\frac{r}{100n}\right)^n$.

(a) State the value of A. (1 mark)

(b) State the value of r. (1 mark)

(c) Calculate V when $n = 4$. (2 marks)

(d) Calculate the effective annual rate of interest when interest is compounded monthly. Give your answer as a percentage correct to **two** decimal places. (3 marks)

Question 17 **(6 marks)**

The recursive formula $T_{n+1} = 1.08T_n$, $T_0 = 2100$ can be used to calculate the value of an investment compounded annually for n years in the Farmers Bank of Western Australia.

(a) What is the annual interest rate? (1 mark)

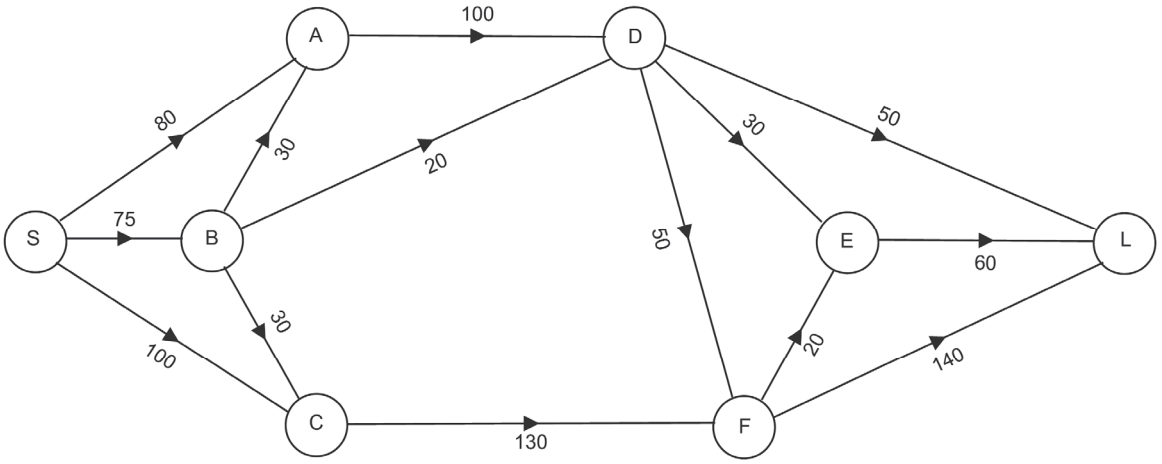
(b) Calculate the value of the investment after seven years. (2 marks)

(c) Determine the interest rate that would produce the same value for the investment above after a time of three years. (3 marks)

See next page

Question 18 **(5 marks)**

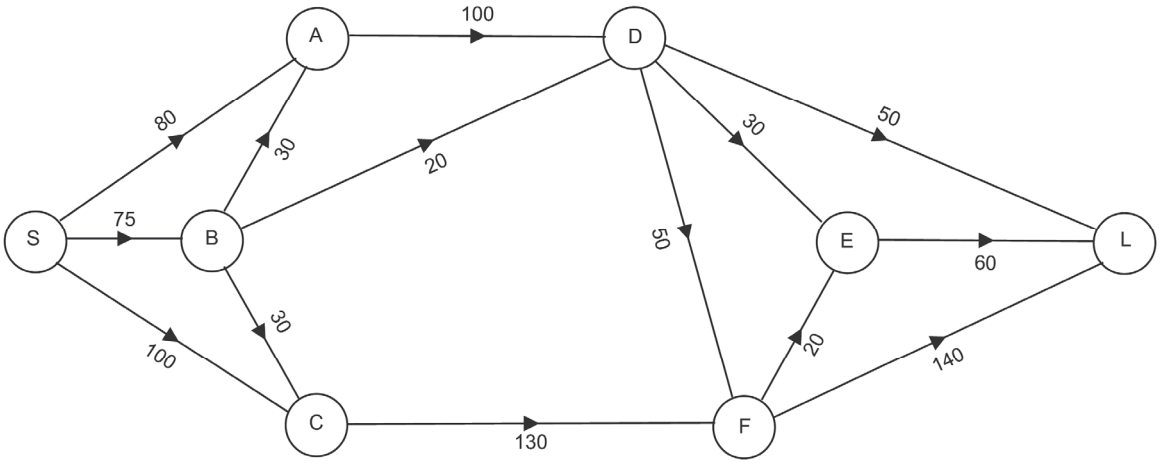
In a mining operation, ore is moved from a central stockpile ‘S’ to a loading station ‘L’ through an ore processing plant consisting of six processors (‘A’, ‘B’, ‘C’, ‘D’, ‘E’ and ‘F’) linked by a system of conveyor belts. The network below displays the operation with the arcs representing the conveyor belts. The number on each arc is the maximum amount of ore, in tonnes per minute, that can be moved along that conveyor belt.



(a) What is the maximum amount of ore, in tonnes per minute, that can be moved from the stockpile S to the loading station L? Show systematic workings to allow your method to be checked. (3 marks)

(b) What effect, if any, would there be on the maximum flow of ore from S to L if the capacity of the conveyor belt FE was increased by 15 tonnes per minute? Justify your answer.

(2 marks)



Question 19 **(11 marks)**

A theatre company performed for three weeks at a large venue capable of seating 4200 people. The attendances, in hundreds, at the evening performances, are shown in the following table.

Table 1: Moving averages

Day		Performance number (p)	Attendance (in hundreds)	Three-point moving average
Week 1	Wednesday	1	20	
	Friday	2	16	20
	Saturday	3	24	22.7
Week 2	Wednesday	4	28	26
	Friday	5	26	29.3
	Saturday	6	34	30.7
Week 3	Wednesday	7	32	31.7
	Friday	8	A	34
	Saturday	9	41	

(a) Calculate the value of the missing entry marked by A. (2 marks)

(b) Given the seasonal index for the Friday performances is 84.64%, calculate the deasonalised data (in hundreds) for Friday of Week 1, correct to **one** decimal place. (2 marks)

Additional information about the theatre attendances is presented in the table below.

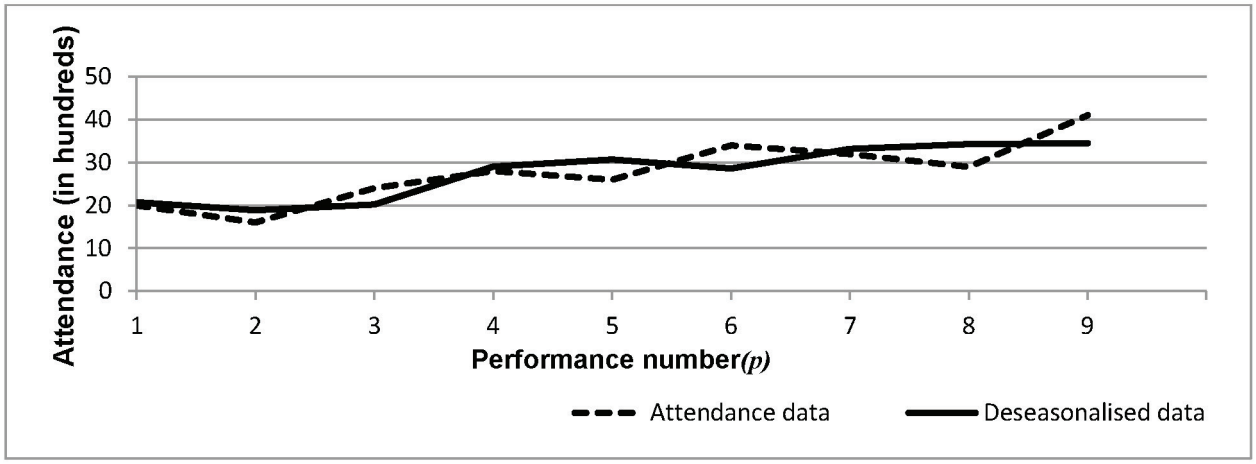
Table 2: Weekly and Percentage Means

		Performance number (p)	Attendance (in hundreds)	Weekly mean	Percentage of weekly mean
Week 1	Wednesday	1	20	20.0	100.0%
	Friday	2	16		80.0%
	Saturday	3	24		120.0%
Week 2	Wednesday	4	28	29.3	95.5%
	Friday	5	26		88.6%
	Saturday	6	34		115.9%
Week 3	Wednesday	7	32	34.0	94.1%
	Friday	8			85.3%
	Saturday	9	41		120.6%

See next page

(c) Show use of the average percentage method to calculate the seasonal index for the Saturday performances. (3 marks)

The attendance data and the deseasonalised data have been graphed on the axes below.



(d) (i) What has been the effect of deseasonalising the attendance data? (1 mark)

(ii) Describe the trend in attendance data over these three weeks. (1 mark)

(iii) Should the company extend its performances into a fifth week? Justify your choice of answer. (2 marks)

Question 20 **(7 marks)**

CensusAtSchool Australia is a nationwide annual project that collects data about students. It provides a snapshot of the characteristics, attitudes and opinions of those students who have completed questionnaires. The tables below present information from the 2010 survey.

Table 1: Number and percentage of students by year level and sex who participated in the 2010 survey

Number of students				
Year level	**Female**	**Male**	**Total**	**% of total**
Yr 4 or below	203	213	416	1.9
Yr 5	1346	1262	2608	11.7
Yr 6	2097	2092	4189	18.8
Yr 7	1466	1464	2930	13.1
Yr 8	2059	1649	3708	
Yr 9	2014	1741	3755	16.3
Yr 10	1461	1197	2658	11.9
Yr 11	793	706	1499	6.7
Yr 12	259	230	489	2.2
Other	32	35	67	
Total	11 730	10 589	22 319	100

Table 2: Favourite takeaway foods by year level, 2010 Survey

Takeaway food	Yr 4 or below	Yr 5	Yr 6	Yr 7	Yr 8	Yr 9	Yr 10	Yr 11	Yr 12	Other
None	2.4	1.4	1.2	1.2	1.3	1.8	1.4	1.1	1.2	1.5
Chicken	6.7	6.8	7.3	6.3	7.3	8.4	9.4	10.1	9.4	14.9
Chips/fries	16.8	16.4	18.0	16.2	17.8	17.4	15.2	13.0	13.5	9.0
Fish	7.2	9.5	8.6	8.1	6.7	6.8	6.3	5.4	5.7	7.5
Fruit/fruit salad	4.3	2.6	2.4	2.4	2.1	2.0	1.8	1.9	2.0	4.5
Hamburgers	12.0	11.8	11.4	11.2	11.0	11.9	10.4	11.3	14.3	6.0
Kebabs/wraps	6.3	6.3	7.5	8.9	9.7	9.9	11.2	13.7	11.5	9.0
Noodle dishes	3.1	3.5	3.2	4.0	4.0	4.4	4.1	5.5	4.9	10.5
Pies/pasties	3.6	3.9	3.3	3.9	3.1	2.6	2.3	2.1	1.8	0.0
Pizza/pasta	22.1	21.6	22.4	22.7	21.9	21.2	23.1	21.4	21.5	11.9
Rice dishes	2.6	3.7	3.6	4.4	5.2	5.1	4.4	6.5	6.5	9.0
Rolls/sandwiches	1.9	1.4	1.5	1.9	1.9	2.2	3.6	2.6	2.5	0.0
Salads	1.2	2.8	2.0	1.7	1.6	1.4	1.8	0.9	1.8	4.5
Other	9.6	8.5	7.5	7.0	6.5	4.9	5.0	4.6	3.3	11.9

* The figures represent the percentages of the total number of students at each year level who participated in the 2010 survey.

See next page

By referring to Table 1 and Table 2, answer the following questions.

(a) Complete Table 1 by adding the missing entries. (1 mark)

(b) Which year level had the highest participation rate in the survey? (1 mark)

(c) What percentage of the students who participated in the survey was female? (1 mark)

(d) Which was the least favoured takeaway food for Year 4 students? (1 mark)

(e) How does this result for Year 4 students compare with that for Year 7 students? (1 mark)

(f) Compare and comment on the popularity of pies/pasties of the students in the early years (up to Year 8) with those of the later years. Use evidence from the table to support your answer. (2 marks)

Question 21 **(8 marks)**

Pollution due to airborne lead has been reduced in Australia because of the requirement, since 1985, that new petrol-powered cars must use unleaded petrol.

The airborne lead data for Perth 1993–1999 are tabulated below.

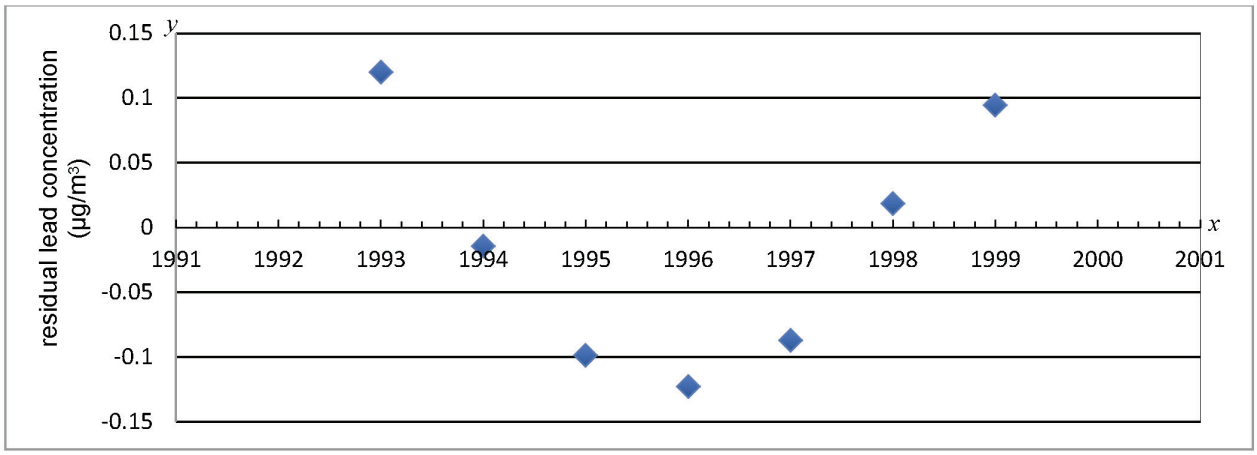
Year (x)	1993	1994	1995	1996	1997	1998	1999
Average concentration of airborne lead (µg/m³) (y)	0.86	0.60	0.39	0.24	0.15	0.13	0.08

(a) State the equation for the least-squares line that models these data. (1 mark)

(b) Determine the correlation coefficient for the data in the table above. (1 mark)

(c) Does the least-squares line determined in part (a) provide a good model for the 1993–1999 data? Justify your answer using evidence from the residual plot below. (2 marks)

Residual plot, annual average airborne lead concentration, Perth, 1993–1999



See next page

(d) A scientist suggested the following recursion model for the Perth lead-concentration data:

$$T_n = 0.6718\ T_{n-1},\ T_1 = 0.86\ ,$$

where $n = 1$ stands for 1993, $n = 2$ stands for 1994, and so on.

Assess whether this recursion model is appropriate for predicting the 2000 lead concentration in Perth air. Give **two** reasons to justify your assessment and state any assumptions that you made in arriving at your conclusion. (4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

ACKNOWLEDGEMENTS

Section Two

Question 10 Data source: Australian Bureau of Statistics. (n.d.). 6401.0 - *Consumer price index, Australia*. Retrieved April 28, 2011, from www.abs.gov.au

Question 12 Data source: *My School* [Table]. (n.d.). Retrieved July, 2011, from www.myschool.edu.au/

Question 15 Data source: World Health Organization. (n.d). *WHO Vaccine Preventable Diseases Monitoring System 2011 global summary.* Retrieved March 17, 2012 from http://apps.who.int/immunization_monitoring/en/globalsummary/countryprofileselect.cfm

Question 20 Australian Bureau of Statistics. (n.d.) *CensusAtSchool Australia* [Tables]. Retrieved April 19, 2011, from www.abs.gov.au/websitedbs/CaSHome.nsf/Home/2010+CensusAtSchool+Summary+Data#. Used under a Creative Commons Attribution 2.5 Australia licence.

Question 21 Data source: Bureau of Infrastructure, Transport and Regional Economics (BITRE). (2005). *Australian Transport Statistics Yearbook 2009*. Retrieved February, 2010, from www.bitre.gov.au/publications/10/files/BITRE_TRANSPORT_STATS_YEARBOOK_2009.pdf

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